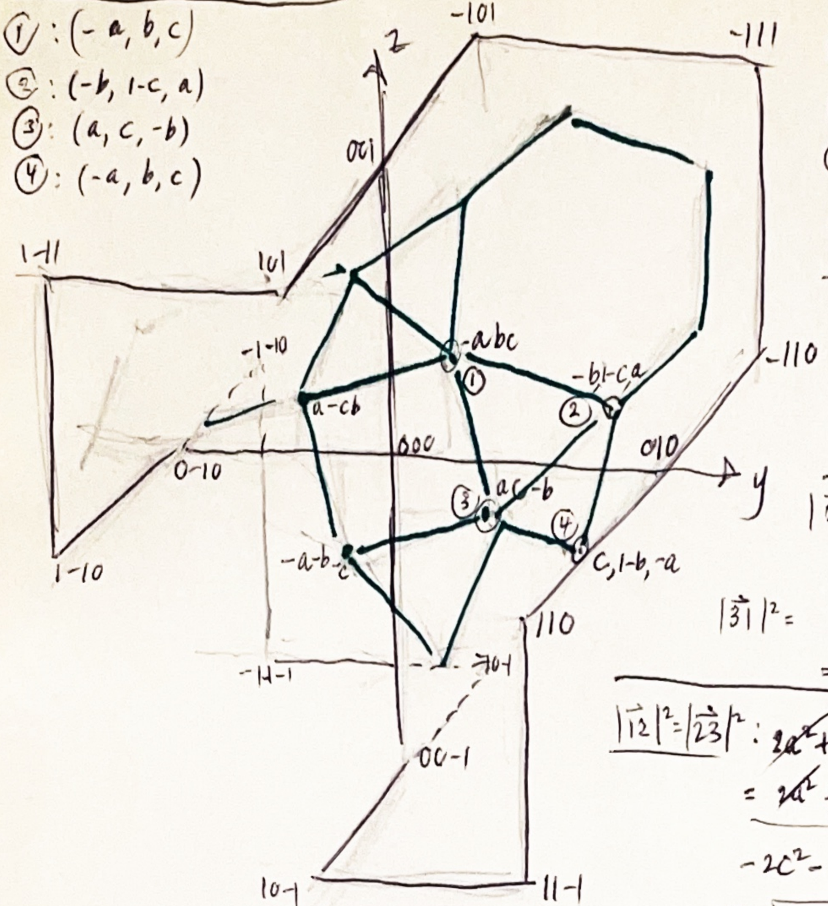
$\boxed{(6.3.4.3^2) \text{ on } D}$ 11/14/98 $(a<b<c)$

(1): $(-a, b, c)$

(2): $(-b, 1-c, a)$

(3): $(a, c, -b)$

(4): $(-a, b, c)$



$\overrightarrow{1 \to 2}$: $(-b+a,\ 1-c-b,\ a-c)$

$\overrightarrow{2 \to 3}$: $(a+b,\ 2c-1,\ -b-a)$

$\overrightarrow{3 \to 1}$: $(-2a,\ b-c,\ c+b)$

<u>Make the triangle equilateral</u>

$|1\to2| = |2\to3| = |3\to1|$

$|\overrightarrow{12}|^2 = (a-b)^2 + (1-b-c)^2 + (a-c)^2$

$= a^2 - 2ab + b^2 + 1 - 2b - 2c + b^2 + 2bc + c^2 + a^2 - 2ac + c^2$

$= \boxed{2a^2 + 2b^2 + 2c^2 + 2bc - 2ac - 2b - 2c + 1}$

$|\overrightarrow{23}|^2$: $a^2 + 2ab + b^2 + 4c^2 - 4c + 1 + a^2 + 2ab + b^2$

$= \boxed{2a^2 + 2b^2 + 4c^2 + 4ab - 4c + 1}$

$|\overrightarrow{31}|^2 = 4a^2 + b^2 - \cancel{2bc} + c^2 + b^2 + \cancel{2bc} + c^2$

$= \boxed{4a^2 + 2b^2 + 2c^2}$

$|\overrightarrow{12}|^2 = |\overrightarrow{23}|^2$: $\cancel{2a^2} + \cancel{2b^2} + 2c^2 + 2bc - 2ac - 2b - 2c + \cancel{1}$

$= \cancel{2a^2} + \cancel{2b^2} + 4c^2 - 4c + 4ab + \cancel{1}$

$-2c^2 - 4ab + 2bc - 2ac - 2b + 2c = 0$

or $\boxed{c^2 + 2ab - bc + ac + b - c = 0}$ (I)

$|\overrightarrow{23}|^2 = |\overrightarrow{31}|^2$: $2a^2 + \cancel{2b^2} + 4c^2 + 4ab - 4c + 1 = 4a^2 + \cancel{2b^2} + 2c^2$

$\boxed{-2a^2 + 2c^2 + 4ab - 4c + 1 = 0}$ (II)

$|\overrightarrow{31}|^2 = |\overrightarrow{12}|^2$ $4a^2 + \cancel{2b^2} + \cancel{2c^2} = 2a^2 + \cancel{2b^2} + \cancel{2c^2} + 2bc - 2ac - 2b - 2c + 1$

$\boxed{2a^2 - 2bc + 2ac + 2b + 2c - 1 = 0}$ (III)

Add II + III: $2c^2 - 2bc + 4ab + 2ac + 2b - 2c = 0$, or $c^2 + 2ab - bc + ac + b - c = 0 \equiv$ (I) just as for $6.3.4.3^2$ on P ★

Hence the three equations I, II, and III are not independent, and one of the three arguments a, b, and c can be chosen arbitrarily.

Choose c, i.e., let c = constant in (I), and then solve for $a = a(b,c)$

(I) → $2ab + ac = -c^2 + bc - b + c$, or $(2b+c)a = -c^2 + bc - b \Rightarrow \boxed{a = \dfrac{-c^2 + bc - b}{2b + c}}$ (I)

Now substitute this value of a in eqns II and III.